

# CUMULANT BASED BLIND EQUALIZATION WITH USER AND CHANNEL IDENTIFICATION FOR MULTIUSER ASYNCHRONOUS DS/CDMA SYSTEMS IN MULTIPATH

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## ABSTRACT

Cumulant based inverse filter criteria for blind equalization of multi-input multi-output (MIMO) systems with non-Gaussian measurements can be applied to blind equalization of multiuser asynchronous DS/CDMA systems in multipath, but each pair of detected symbol sequence and estimated overall multipath channel (convolution of multipath channel and spreading code) cannot be identified with the associated active user. In this paper, a user identification algorithm (UIA) is proposed that further processes each estimated overall channel response by a bank of matched filters of user signature sequences followed by a comparator to identify each detected symbol sequence with the associated user. Then some simulation results are presented to support the efficacy of the proposed UIA. Finally, some conclusions are provided.

## 1. INTRODUCTION

Recently, direct sequence code division multiple access (DS/CDMA) techniques have proven effective to wireless communications. It has been widely known that for a multiuser asynchronous DS/CDMA system with  $K$  active users, the baseband discrete-time received signal  $y[n]$  at chip sampling rate  $1/T_c$  for the base station (reverse-link) can be modeled as [1-3]

$$y[n] = \sum_{j=1}^K \sum_{k=-\infty}^{\infty} u_j[k] h_j[n - kP] + w[n] \quad (1)$$

where  $u_j[n]$  is the symbol sequence transmitted by user  $j$ ,  $w[n]$  is additive white Gaussian noise and

$$h_j[n] = c_j[n] * g_j[n] = \sum_{k=0}^{P-1} c_j[k] g_j[n - k] \quad (2)$$

in which  $g_j[n]$  and  $c_j[n]$  are the multipath fading channel and the signature sequence of length  $P$  (a binary pseudo-noise sequence of  $\pm 1$ ) associated with user  $j$ , respectively. The received signal  $y[n]$  can also be expressed as the following multi-input multi-output (MIMO) linear time-invariant (LTI) system [1-3]:

$$\begin{aligned} \mathbf{y}[n] &= \mathbf{H}[n] * \mathbf{u}[n] + \mathbf{w}[n] \\ &= \sum_{k=-\infty}^{\infty} \mathbf{H}[k] \mathbf{u}[n - k] + \mathbf{w}[n] \end{aligned} \quad (3)$$

where

$$\begin{aligned} \mathbf{y}[n] &= (y[nP], y[nP + 1], \dots, y[nP + P - 1])^T \\ \mathbf{u}[n] &= (u_1[n], \dots, u_K[n])^T \\ \mathbf{w}[n] &= (w[nP], w[nP + 1], \dots, w[nP + P - 1])^T \end{aligned}$$

and  $\mathbf{H}[n]$  is a  $P \times K$  impulse response matrix with the  $(i, j)$ th element equal to

$$[\mathbf{H}[n]]_{ij} = h_j[nP + i - 1] \quad (4)$$

Assume that  $\mathbf{V}[n]$  is a  $K \times P$  inverse filter and

$$\mathbf{e}[n] = (e_1[n], e_2[n], \dots, e_K[n])^T = \mathbf{V}[n] * \mathbf{y}[n] \quad (5)$$

Cumulant based inverse filter criteria [4-7] for blind equalization of MIMO systems with only non-Gaussian measurements  $\mathbf{y}[n]$  are potentially applicable to the estimation of  $\{u_j[n], h_j[n]\}$  due to the following fact:

- (F1) The optimum  $\{e_l[n], \bar{h}_l[n]\}$ ,  $l = 1, 2, \dots, K$  obtained by cumulant based inverse filters are an unknown permutation of  $\{\alpha_j u_j[n - \tau_j], h_j[n + \tau_j] / \alpha_j\}$ ,  $j = 1, 2, \dots, K$  where  $\alpha_j$  and  $\tau_j$  are unknown scale factors and time delays, respectively.

In this paper, a user identification algorithm (UIA) is proposed to identify  $\{e_l[n], \bar{h}_l[n]\}$ ,  $l = 1, 2, \dots, K$  with the associated  $K$  active users, respectively. Next, let us present the proposed UIA for multiuser asynchronous DS/CDMA systems in multipath with user signature sequences known in advance.

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## 2. USER IDENTIFICATION ALGORITHM

With  $\{e_l[n], \bar{h}_l[n]\}$ ,  $l = 1, 2, \dots, K$  provided by cumulant based inverse filters and the signature sequence set

$$\mathcal{C} = \{c_k[n], k = 1, 2, \dots, N\}$$

known *a priori*, the proposed UIA is based on the following two facts:

- (F2) Assume that a given complex sequence  $b[n]$  is input to a complex allpass system  $c[n]$  and  $a[n] = b[n] * c[n]$  is the corresponding output sequence. Let  $|A(\omega)|$  and  $\phi_a(\omega) = \arg\{A(\omega)\}$  denote the magnitude and the phase spectra of  $a[n]$  with linear phase (time delay) removed, respectively. Define

$$\Lambda(a[n]) = \left| \sum_{n=-\infty}^{\infty} (a[n])^p (a^*[n])^q \right| \quad (6)$$

and

$$\mathcal{F}(a[n]) = \int_{-\pi}^{\pi} |A(\omega)| [\phi_a(\omega)]^2 d\omega \quad (7)$$

where  $p + q \geq 3$ . Chien, Yang and Chi [8] have shown that the smaller  $\mathcal{F}(a[n])$ , the larger  $\Lambda(a[n])$  with  $\mathcal{F}(a[n]) = 0$  for  $\phi_a(\omega) = 0$  (i.e.,  $\phi_c(\omega) = -\phi_b(\omega)$ ) when  $a[n]$ ,  $b[n]$  and  $c[n]$  are real. It can be shown that this statement is also true when  $a[n]$ ,  $b[n]$  and  $c[n]$  are complex.

- (F3) Each signature sequence  $c_k[n] \in \mathcal{C}$  is a pseudo-random (approximate allpass) sequence with autocorrelation function close to a discrete-time delta function and uncorrelated with  $c_l[n] \in \mathcal{C}$  for  $k \neq l$ . In other words, the phase  $\phi_{c_l}(\omega)$  is approximately random and uncorrelated with  $\phi_{c_k}(\omega)$  and  $c_l[n] = c_k[n]$  when  $\phi_{c_l}(\omega) = \phi_{c_k}(\omega)$ .

The  $\{e_l[n], \bar{h}_l[n]\}$  is identified with the active user  $j$  (with the signature sequence  $c_j[n]$ ) for which

$$\Lambda(a_{l,j}[n]) = \max \{\Lambda(a_{l,k}[n]), k = 1, 2, \dots, N\} \quad (8)$$

where

$$a_{l,k}[n] = \bar{h}_l[n] * c_k[-n] \quad (9)$$

in which  $c_k[n] \in \mathcal{C}$ . The proposed UIA given by (8) and (9) can be easily implemented by an FIR filter bank of  $N$  matched filters  $c_k[-n]$  followed by a comparator as shown in Figure 1. Next, let us present how (F2) and (F3) lead to the proposed UIA.

It can be easily inferred by (F3) that

$$\begin{aligned} \mathcal{F}(a_{l,k}[n]) &\simeq \int_{-\pi}^{\pi} |G_j(\omega)| [\phi_{g_j}(\omega) + \phi_{c_j}(\omega) \\ &\quad - \phi_{c_k}(\omega)]^2 d\omega \quad (\text{by (2), (9) and (7)}) \\ &= \mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_3 \end{aligned} \quad (10)$$

where

$$\mathcal{F}_1 = \int_{-\pi}^{\pi} |G_j(\omega)| [\phi_{c_j}(\omega) - \phi_{c_k}(\omega)]^2 d\omega \quad (11)$$

$$\mathcal{F}_2 = \int_{-\pi}^{\pi} |G_j(\omega)| [\phi_{g_j}(\omega)]^2 d\omega \quad (12)$$

$$\mathcal{F}_3 = 2 \int_{-\pi}^{\pi} |G_j(\omega)| \phi_{g_j}(\omega) \cdot [\phi_{c_j}(\omega) - \phi_{c_k}(\omega)] d\omega \quad (13)$$

Note that  $\mathcal{F}_2$  is basically a constant and that  $\phi_{g_j}(\omega)$  is smooth compared to  $\phi_{c_k}(\omega)$  for all  $k$  which implies that  $\mathcal{F}_1 \gg \mathcal{F}_3$  when  $\phi_{c_j}(\omega) \neq \phi_{c_k}(\omega)$  by (F3). Therefore,  $\mathcal{F}(a_{l,k}[n])$  is minimum when  $c_k[n] = c_j[n]$ , and thus  $\Lambda(a_{l,k}[n])$  is maximum for  $c_k[n] = c_j[n]$  by (F2).

Three remarks regarding the proposed UIA given by (8) and (9) are worth mentioning as follows:

- (R1) The same identified user  $j$  will be obtained for either of  $\bar{h}_l[n]$  and  $\alpha \bar{h}_l[n - \tau]$  because

$$\Lambda(a_{l,k}[n]) = |\alpha|^{-(p+q)} \Lambda(\alpha a_{l,k}[n - \tau]) \quad (14)$$

by (6), (8) and (9).

- (R2) After  $\bar{h}_l[n]$  is correctly identified with  $c_j[n]$  (i.e.,  $\bar{h}_l[n] = h_j[n]$ ), the optimum

$$\begin{aligned} a_{l,j}[n] &= h_j[n] * c_j[-n] = g_j[n] * c_j[n] * c_j[-n] \\ &\simeq g_j[n] * P\delta[n] = Pg_j[n] \end{aligned} \quad (15)$$

also provides an estimate of the multipath channel  $g_j[n]$ . Moreover, the least-squares (LS) estimate  $\hat{g}_j[n]$  can be obtained from  $h_j[n]$  which is linearly related to  $g_j[n]$  by (2).

- (R3) In conjunction with successive cancellation type blind deconvolution algorithms such as MIMO inverse filter criteria reported in [4-7], each user identification can be performed successively with  $\sum_n |\bar{h}_l[n]|^2$  normalized to a constant for all  $l$ . Then the number of active users can be determined to be  $\mathcal{K} \leq N$  when the maximum  $\Lambda(a_{\mathcal{K}+1,j}[n])$  (see (8)) is below a threshold.

## 3. SIMULATION RESULTS

Let us show some simulation results to support the efficacy of the proposed UIA. Assume that the spreading sequence set  $\mathcal{C}$  includes  $N = P + 2$  Gold codes  $c_j[n]$  of length  $P$ , and each multipath channel  $g_i[n]$  is a Rayleigh fading channel of  $L$  paths as follows:

$$g_i[n] = \sum_{k=0}^{L-1} b_{i,k} e^{j\varphi_{i,k}} \delta[n - k] \quad (16)$$

where  $\varphi_{i,k}$ ,  $k = 0, 1, \dots, L - 1$  are independent identically distributed (i.i.d.) random variables uniformly distributed over  $[-\pi, \pi)$ , and  $b_{i,k}$ ,  $k = 0, 1, \dots, L - 1$  are i.i.d. Rayleigh distributed random variables with  $E[|b_{i,k}|^2] = \Phi(kT_c)$  where  $\Phi(\tau)$  is the multipath delay profile given by [9]

$$\Phi(\tau) = e^{-\tau/T_c} \quad (17)$$

For simplicity, in the simulation synthetic  $\bar{h}_j[n]$  (without using cumulant based inverse filter criteria) were generated by

$$\bar{h}_j[n] = h_j[n] + v_j[n] = c_j[n] * g_j[n] + v_j[n] \quad (18)$$

where  $\{l_1, l_2, \dots, l_K\}$  was a permutation of  $\{1, 2, \dots, K\}$ , and  $v_j[n]$  were i.i.d. complex Gaussian with zero mean and variance  $\sigma_v^2$  that were treated as estimation errors generated by cumulant based inverse filter criteria. The proposed UIA with  $p = q = 2$  was employed to identify  $\bar{h}_j[n]$  with  $h_j[n]$  (or  $c_j[n]$ ),  $j = 1, 2, \dots, K$  for channel estimation error variance  $\sigma_v^2$  ranging between 0 and 1.

The simulation was performed for three different code lengths  $P = 31, 63$  and  $127$ , respectively. The number of active users was  $K = 30$  and each  $g_j[n]$  was a 3-path ( $L = 3$ ) channel generated using (16). One hundred realizations were performed and then user identification error rate (UIER) defined as

$$\text{UIER} = \frac{1}{100 \times K} \left( \sum_{i=1}^{100} E_i \right) \times 100\% \quad (19)$$

was calculated as performance index where  $E_i$  is the number of wrong identifications out of  $K$  active users at the  $i$ th realization.

Table 1 shows some simulation results UIER (%) for the above three cases of  $P$ . One can see, from this table, that UIER is smaller for either smaller  $\sigma_v^2$  or larger  $P$ , and that UIER = 0 (perfect user identification) happens in many cases. These simulation results justify the good performance of the proposed user identification algorithm.

#### 4. CONCLUSIONS

In this paper, a UIA shown in Figure 1 was presented for the user identification of multiuser asynchronous DS/CDMA systems assuming that only user signature sequences are known in advance. In conjunction with the proposed UIA, cumulant based inverse filter criteria such as those reported in [4-7] can be applied to blind equalization for multiuser asynchronous DS/CDMA systems in multipath. Some simulation results were presented to justify the good performance of

the proposed UIA. As a final remark, the proposed UIA can be easily implemented by software and is suitable for software radio.

#### 5. REFERENCES

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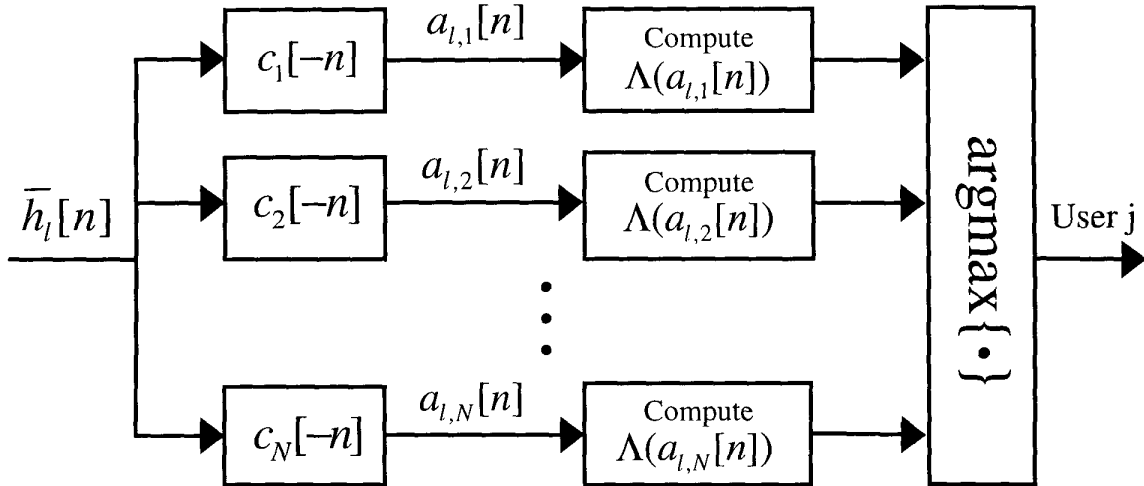


Figure 1. The proposed user identification algorithm (UIA)

Table 1. Simulation results: UIER (%) for  $P=31, 63$  and  $127$ , respectively.

UIER(%)											
$P \backslash \sigma_v^2$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
31	0	0	0.20	1.10	2.67	4.40	6.43	8.33	9.73	11.5	13.3
63	0	0	0	0.20	0.77	1.23	2.00	2.70	3.60	4.43	5.20
127	0	0	0	0	0	0.07	0.13	0.30	0.67	1.03	1.53